# DEPARTMENT OF MATHEMATICS Ph.D., COURSE WORK

Semester	Paper Code	Course Title	Credits	Total	
Core Courses					
I	17MATR0101	Advances in Algebra	4		
	17MATR0102	Advances in Analysis	4		
	17MATR0103	Advances in Differential Equations	4		
	17MATR0104	Research Methodology	4		
II	17MATR0205	Advances in Numerical Methods	4		
	17MATR02SX	Specific course to be prescribed by the	4	24	
		Doctoral Committee		24	
	Seminar (3)				
	Term Paper/Topical Research				
	a) Project Planning including literature collection,		4		
III	finalisation of objectives and methodology				
Semester	b) Field / Lab Studies, Data collection, compilation 32				
onwards	of results, statistical analysis, results and final				
	conclusion				
End of	Synopsis and thesis submission, final viva 6				
Program					

# List of courses that are candidate centric (17MATR02SX)

17MATR02S1	Mathematical Analysis
17MATR02S2	Discrete Mathematics
17MATR02S3	Mathematical Modeling
17MATR02S4	Soft Computing

# 17MATR0101

# **ADVANCES IN ALGEBRA**

Credits: 4

Learning Objectives: To impart some advances in Commutative algebra and Extension fields.

**Learning Outcomes:** The learner will be able to

- understand the properties of different types of ideals;
- recognize the concept of a module and their constructions;
- understand the properties of modules of fractions;
- recognize the properties of tensor product of algebras;
- understand the Extension fields, their types and characterizations.

**Unit-1:** Rings and Ideals: Rings and ring homomorphisms – Ideals, Quotient rings – Zerodivisors, Nilpotent elements, Units – Prime ideals and maximal ideals – Nilradical and Jacobson radical – Operations on ideals – Extension and contraction.

**Unit-2:** Modules: Modules and module homomorphisms – Submodules and Quotient Modules – Operations on Submodules – Direct sum and product- Finitely Generated Modules - Exact Sequences.

**Unit-3:** Tensor Product of Modules – Restriction and extension of scalars – Exactness Properties of the Tensor Product – Algebras and Tensor Product of Algebras.

**Unit -4:** Rings and Modules of Fractions: Local Properties – Extended and Contracted Ideals in Rings of Fractions.

**Unit-5:** Field Theory: Basic Theory of Field Extensions - Algebraic Extensions - Splitting Fields and Algebraic Closures - Separable and Inseparable Extensions.

### References:

- 1. M.F. Atiyah and I.G. MacDonald, An Introduction To Commutative Algebra , Addison-Wesley Series in Mathematics, 1969.
- 2. David S. Dummit and Richard M. Foote, Abstract Algebra, Third Edition, John Wiley & Sons, Inc., 2004.
- 3. Arthur Allen Altman and Steven Kleiman, A Term of Commutative Algebra, Worldwide Center of Mathematics, 2013.
- 4. H. Matsumura, Commutative ring theory, Cambridge Studies in Advanced Mathematics, Cambridge University Press, 1986
- 5. Steven Roman, Field theory, Springer Science & Business Media, 2005.

- 6. A. V. Jayanthan, Introduction to Commutative Algebra, NPTEL Courses, <a href="http://nptel.ac.in/courses/111106098/">http://nptel.ac.in/courses/111106098/</a>
- 7. Jugal K. Verma, , Algebra II, NPTEL Courses, http://nptel.ac.in/courses/111101001/
- 8. Sudhir R. Ghorpade, Lectures on Commutative Algebra, <a href="http://www.math.iitb.ac.in/~srg/Lecnotes/afspune\_des.html">http://www.math.iitb.ac.in/~srg/Lecnotes/afspune\_des.html</a>

### 17MATR0102

# ADVANCES IN ANALYSIS

Credits: 4

**Learning Objectives:** To impart some advances in Commutative algebra and Extension fields.

**Learning Outcomes:** The learner will be able to

- understand the properties of different types of ideals;
- recognize the concept of a module and their constructions;
- understand the properties of modules of fractions;
- recognize the properties of tensor product of algebras;
- understand the Extension fields, their types and characterizations.

**Unit-1:** Compact Operators: Some characterizations – Space of compact operators – Further properties.

**Unit-2:** Spectral Results for Banach Space Operators : Eigen spectrum and approximate Eigen spectrum.

**Unit-3:** Spectrum and resolvent set: Spectral radius – Spectral mapping theorem –More results based on resolvent.

**Unit-4:** Operators on Hilbert spaces: Adjoint of an operator – Compactness of the adjoint operator – Sesquilinear functionals – Integration of operator-valued functions revisited.

**Unit-5:** Complete Metric Spaces and Function Spaces : Complete metric spaces - A Space Filling curve - Compactness in metric spaces

### References:

- 1. M. Thamban Nair, Functional Analysis- A First Course, Prentice Hall of India Private Limited, New Delhi, 2008.
- 2. James R. Munkres, Topology, Second edition, PHI Learning Limited, New Delhi, 2012.
- 3. C. Goffman and G. Pedrick, First Course in Functional Analysis, Prentice-Hall of India, New Delhi, 1995.
- 4. B. V. Limaye, Functional Analysis, Wiley Eastern, New Delhi, 1981.
- 5. S. Willard, General Topology, Addison-Wesley Publishing Company, Inc., Reading, Mass, 1970.

- 6. C. Wayne Patty, Foundations of Topology, Jones and Bartlett Publishers, 2009.
- 7. Richard Melrose, Introduction to functional Analysis, MIT OpenCourseWare, <a href="https://ocw.mit.edu/courses/mathematics/18-102-introduction-to-functional-analysis-spring-2009/lecture-notes/">https://ocw.mit.edu/courses/mathematics/18-102-introduction-to-functional-analysis-spring-2009/lecture-notes/</a>
- 8. P. Veeramani, Topology, NPTEL Courses, <a href="http://nptel.ac.in/courses/111106054/1">http://nptel.ac.in/courses/111106054/1</a>

# 17MATR0103 ADVANCES IN DIFFERENTIAL EQUATIONS Credits: 4

**Learning Objectives:** To impart some advances in Commutative algebra and Extension fields.

**Learning Outcomes:** The learner will be able to

- understand the properties of different types of ideals;
- recognize the concept of a module and their constructions;
- understand the properties of modules of fractions;
- recognize the properties of tensor product of algebras;
- understand the Extension fields, their types and characterizations.

**Unit -1:** First and second order Linear Equations: First order Equations—Exact Differential Equations—Second order linear equations—Partial differential equations and ODE.

**Unit -2:** General theory of initial value problems: Introduction—Sufficient condition for uniqueness of solution—Sufficient condition for existence of solution—Continuous dependence of the solution on initial data and dynamics—Continuation of a solution into larger intervals and maximal interval of existence—Existence and uniqueness of a system of equations.

**Unit-3:** Linear systems and qualitative analysis: General  $n^{th}$  order equations and linear systems – Autonomous homogeneous systems – Two-dimensional systems – Stability analysis – Higher dimensional systems – Invariant subspaces under the flow  $e^{tA}$  – Non-homogeneous, Autonomous systems.

**Unit-4:** Qualitative theory: Introduction – General definitions and results – Lyapunov stability, Lyapunov function – Invariant subspaces and manifolds – Phase plane analysis – Periodic orbits.

**Unit-5:** Two point boundary value problems: Introduction – Linear problems – General second order equations.

# References:

- 1. A. K. Nandakumaran, P. S. Datti& Raju K. George, Ordinary differential equations: Principles and Applications, Cambridge University Press, 2017.
- 2. Website: http://nptel.ac.in/syllabus/111108081/
- 3. Lawrence Perko, Differential Equations and Dynamical systems, Springer-Verlag, New-York, 2001.

- 4. Chi Y. Lo, Boundary Value Problems, Allied-Publishers Pvt Ltd, New Delhi, 2003
- 5. E. A. Coddington and N. Levinson, Theory of ordinary Differential Equations, Tata-McGraw Hill, 1972.
- 6. M. W. Hrisch, S. Smale and R. L. Devaney, Differential Equations, Dynamical Systems & An Introduction to Chaos, Academic Press, 2004.
- 7. E. L. Ince, Ordinary Differential Equations, Dover, 1956.
- 8. S. Lefschetz, Differential Equations: Geometric Theory, Dover, 1977.
- 9. G. F. Simmons, Differential Equations with Applications and Historical Notes, Tata-McGraw Hill, 1991.
- 10. G. F. Simmons and S. G. Krantz, Differential Equations; Theory, Techniques and Practice, Tata-McGraw Hill, 2007.

# 17MATR0104

# RESEARCH METHODOLOGY

Credits: 4

**Learning Objectives:** To impart some advances in Commutative algebra and Extension fields.

**Learning Outcomes:** The learner will be able to

- understand the properties of different types of ideals;
- recognize the concept of a module and their constructions;
- understand the properties of modules of fractions;
- recognize the properties of tensor product of algebras;
- understand the Extension fields, their types and characterizations.

**Unit-1:** Introduction – Meaning – Objectives – Motivation – Types – Research Approache – Significance – Methodology – Research and Scientific Method – Defining the Problem – Selecting the problem – Necessity – Technique involved in defining a problem – Research Design – Meaning – Need for Research Design – Features of a good Design – Important concepts – Different Research Designs.

**Unit-2:** Sampling Design – Census and Sample Survey – Implications of a Sample Design – Steps in Sampling Design – Criteria of Selecting a sampling procedure – Characteristics – Types – Measurement and Scaling Techniques – Measurement in Research – Measurement Scales – Sources of Error in Measurement – Technique of Developing Measurement Tools

**Unit-3:** Methods of Data Collection – Collection of Primary Data – Observation Method – Interview Method – Collection of Data through Questionnaires and Schedules – Difference between Questionnaires and Schedules – Some other Methods – Collection of Secondary data – Selection of Appropriate Method.

**Unit-4:** Processing and Analysis of Data – Processing Operations – Some Problems in Processing – Elements/Types of Analysis – Statistics – Measures of Central Tendency, Dispersion, Skewness and Relationship –Sampling Fundamentals – Need for Sampling – Fundamental Definitions – Important Sampling Distributions

**Unit-5:** Interpolation and Report writing- Meaning of interpolation- why interpolation- Techniques of Interpolation- Precaution in Interpolation-Significance of Report writing- Different Steps in writing Report- Layout of the Research Report- Types of Reports- The Computer- Introduction\_ The computer and computer technology- The computer system-Important Characteristics.

# References:

- 1. C. R. Kothari, "Research Methodology Methods and Techniques", Second Edition, New age International (P) Ltd, Publishers, New Delhi, 2004
- 2. Ackoff, Russell., The Design of Social Research, Chicago: University of Chicago press, 1961
- 3. Bailey, Kenneth D., "Methods of social Research," New York, 1978.
- 4. Denzin, Norman, The research Act, Chicago: Aldine, 1973

# 17MATR0205 ADVANCES IN NUMERICAL METHODS Credits: 4

**Learning Objectives:** To impart some advances in Commutative algebra and Extension fields.

**Learning Outcomes:** The learner will be able to

- understand the properties of different types of ideals;
- recognize the concept of a module and their constructions;
- understand the properties of modules of fractions;
- recognize the properties of tensor product of algebras;
- understand the Extension fields, their types and characterizations.

**Unit-1:** Solutions of Eigen – equations: Concept of Eigen-System- Polynomial Method- The Fadeev- Liverier Method- Graeff's Root Squaring Method for finding Roots of Polynomial

**Unit-2:** Power Method to find Eigen value and Eigen Vector- QR Iterative Method Matrix Eigen value Problem: Power Method- Schur's and Gershgorin's Theorems-Orthogonal Factorizations and Least –Squares Problems

**Unit-3:** Boundary value problems: Finite Difference Method- Functions values at End points- Derivative conditions at End points- Shooting Method- Function values at End points- Derivative values at end points- Shooting Method for Non-linear equations- function values at end point- derivative values at end point- Numerical solutions of Ordinary Differential equation- Milne's Predictor corrector method- Adams Bashforth predictor and corrector Method

**Unit-4:** Two point boundary value problems- Finite Difference Method- The Linear Problem with Dirichlet boundary conditions -The Linear problem with Non Dirichlet Boundary Conditions- Finite Difference Method- Non linear problems- The Shooting Method –Linear Boundary value problems

**Unit-5:** Numerical Solutions of Partial Differential Equations: Introduction –Difference quotients- Geometrical representation of partial differential quotient- Classification of Partial differential equations- Elliptic equations-Solutions to Laplace's equation by Liebmann's iteration process- Poisson's equation and its solutions- parabolic equations- Crank Nicholson method- Hyperbolic equations

# References:

- 1. Numerical Computational Methods- P.B. Patil, U.P. Verma Narosa Publishing House Pvt. Ltd, 2006
- 2. Numerical Analysis- Mathematics of Scientific Computing IIIrd Edition by David Kincaid and Ward Cheney, American Mathematical Society, Providence, Rhode Island, 2010
- 3. An Introduction to Numerical Analysis Third Edition Devi Prasad Narosa Publishing House Pvt. Ltd., 2006
- 4. A Friendly Introduction to Numerical Analysis by Brain Bradie Christopher Newport University, Pearson Prentice Hall, 2006
- 5. V.N.Vedamurthy and N.Ch.S.N.Iyengar, Numerical Methods, Vikas Publishing house, Pvt. Ltd, 1998
- 6. R.L.Burden and J.Douglas Faires, Numerical Analysis, Thompson Books, USA, 2005
- 7. Curtis. F. Gerald, Patrick & O.Wheatley, Applied Numerical Analysis, 5<sup>th</sup> Edition, Pearson Education, New Delhi, 2005